LIMITS of FUNCTIONS, CH1 (1.1-1.4)
Concepts/Skills to know:

- Distinguish between general **function *f(x)*** as general rule/expression and graphical representation versus a specific **function-value *f(x)*** at a specific **x-value**.
- Understand and use notation for:
  - limit of a function $\lim_{x\to a} f(x) = L$ — ***f(x)*** approaches ***L*** as **x** approaches ***a*** from both sides
  - left-hand limit of a function $\lim_{x\to a^-} f(x) = L$ — ***f(x)*** approaches ***L*** as **x** approaches ***a*** from the left side
  - right-hand limit of a function $\lim_{x\to a^+} f(x) = L$ — ***f(x)*** approaches ***L*** as **x** approaches ***a*** from the right side
- Make a table for values close to the limit value & also interpret the graphical representation of the limit.
- Use the following strategies to find limits:
  - Evaluate limits by direct **substitution**, if possible and if defined.
  - If limit of ***f(x)*** as ***x*** approaches ***a*** cannot be evaluated by direct substitution, then use algebra to manipulate, **factor** and **simplify** expression, and <u>*then*</u> evaluate by direct **substitution**.
  - Use a graph and/or table of values to verify the limit that was found.
- Know the formal **definition of limit** of a function and use the definition to find δ, given any ε:

  $\lim_{x\to a} f(x) = L$ means that for any ε>0 there exists δ>0 such that

  if $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$ (ε and δ are very small values)
- Show that $(a-\delta) < x < (a+\delta)$ is equivalent to $0 < |x-a| < \delta$ on the x-axis

  & that $(L-\varepsilon) < f(x) < (L+\varepsilon)$ is equivalent to $|f(x) - L| < \varepsilon$ on the y-axis.
- Use the following Limit Theorems:

Constant function:
$$\lim_{x\to a} c = c$$

Simple Linear function:
$$\lim_{x\to a} x = a$$

Sum or Difference:
$$\lim_{x\to a}\left[f(x) \pm g(x)\right] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

Product:
$$\lim_{x\to a}\left[f(x)\cdot g(x)\right] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

Quotient:
$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

Scalar:
$$\lim_{x\to a}\left[c\cdot f(x)\right] = c\cdot\left[\lim_{x\to a} f(x)\right]$$

General Linear function:
$$\lim_{x\to a}(mx+b) = ma+b$$

Simple Power function:
$$\lim_{x\to a} x^n = a^n$$

Power of a function:
$$\lim_{x\to a}\left[f(x)\right]^n = \left[\lim_{x\to a} f(x)\right]^n$$

Polynomial function:
$$\lim_{x\to a} f(x) = f(a)$$

Rational function:
$$\lim_{x\to a} q(x) = q(a)$$

Radical (root) function:
$$\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a} \quad or \quad \lim_{x\to a} x^{\frac{1}{n}} = a^{\frac{1}{n}}$$

Power of a radical (root):
$$\lim_{x\to a}(\sqrt[n]{x})^m = \left(\sqrt[n]{a}\right)^m \quad or \quad \lim_{x\to a} x^{\frac{m}{n}} = a^{\frac{m}{n}}$$

Root of a function:
$$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$$

- Find limits involving infinity & know that ∞ is <u>not</u> a real number:

limit does <u>not</u> exist where L→±∞: (look at function behavior)
$$\lim_{x\to a^-} f(x) = \pm\infty \qquad \lim_{x\to a+} f(x) = \pm\infty \qquad \lim_{x\to a} f(x) = \pm\infty$$
(***a*** is a real number, ***L*** is <u>not</u> a real number)

limit may exist where x→±∞: (find function-value)
$$\lim_{x\to -\infty} f(x) = L \qquad \lim_{x\to +\infty} f(x) = L$$
(***a*** is <u>not</u> a real number, ***L*** <u>is</u> a real number)

- Use Limit Theorems: $\lim_{x\to +\infty} \frac{c}{x^k} = 0$ and $\lim_{x\to -\infty} \frac{c}{x^k} = 0$ (**k** is a positive rational number, **c** is any real number)